## Chapter 2. TENSION MEMBER DESIGN

### 2.1 INTRODUCTORY CONCEPTS

- Stress: The stress in an axially loaded tension member is given by Equation (4.1)

$$f = \frac{P}{A} \hspace{4cm} (4.1)$$

  where, P is the magnitude of load, and

  A is the cross-sectional area normal to the load

- The stress in a tension member is uniform throughout the cross-section except:

  - near the point of application of load, and

  - at the cross-section with holes for bolts or other discontinuities, etc.

- For example, consider an 8 x ½ in. bar connected to a gusset plate and loaded in tension as shown below in Figure 4.1
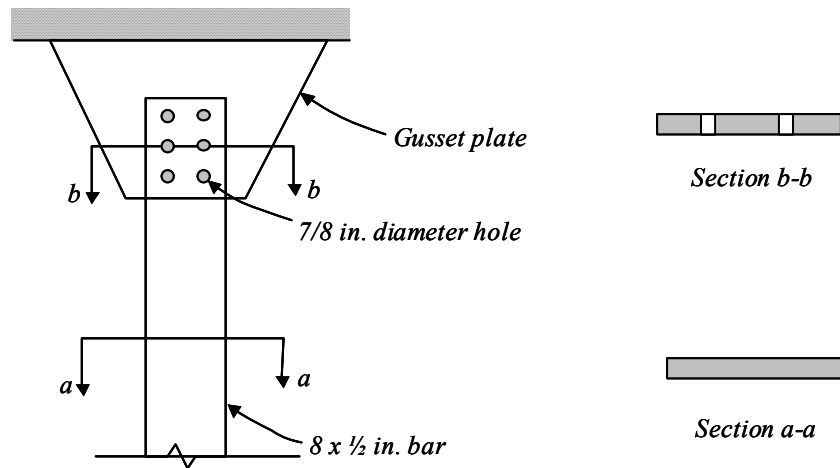


**Figure 4.1** Example of tension member.

- Area of bar at section $a - a = 8$ x ½ $= 4$ in$^2$

- Area of bar at section $b - b = (8 - 2$ x 7/8 $)$ x ½ $= 3.12$ in$^2$

- Therefore, by definition (Equation 4.1) the reduced area of section *b – b* will be subjected to higher stresses

- However, the reduced area and therefore the higher stresses will be <u>localized</u> around section b – b.

- The unreduced area of the member is called its gross area = $A_g$

- The reduced area of the member is called its net area = $A_n$

## 2.2 STEEL STRESS-STRAIN BEHAVIOR

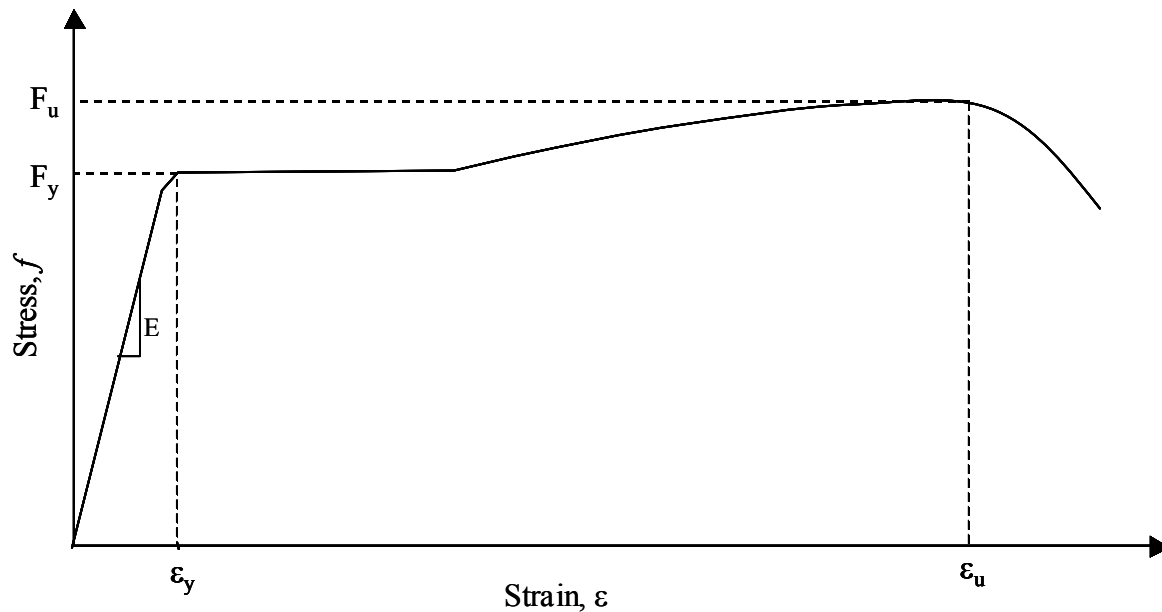- The stress-strain behavior of steel is shown below in Figure 4.2



**Figure 4.2** Stress-strain behavior of steel

- In Figure 4.2, E is the elastic modulus = 29000 ksi.

  $F_y$ is the yield stress and $F_u$ is the ultimate stress

  $\varepsilon_y$ is the yield strain and $\varepsilon_u$ is the ultimate strain

- Deformations are caused by the strain ε. Figure 4.2 indicates that the structural deflections will be small as long as the material is elastic ($f < F_y$)

- Deformations due to the strain ε will be large after the steel reaches its yield stress $F_y$.

## 2.3 DESIGN STRENGTH

- A tension member can fail by reaching one of two limit states:

  (1) excessive deformation; or (2) fracture

- Excessive deformation can occur due to the yielding of the gross section (for example section a-a from Figure 4.1) along the length of the member

- Fracture of the net section can occur if the stress at the net section (for example section b-b in Figure 4.1) reaches the ultimate stress $F_u$.

- The objective of design is to prevent these failure before reaching the ultimate loads on the structure (*Obvious*).

- This is also the load and resistance factor design approach recommended by AISC for designing steel structures

### 2.3.1 Load and Resistance Factor Design

The load and resistance factor design approach is recommended by AISC for designing steel structures. It can be understood as follows:

Step I. Determine the ultimate loads acting on the structure

- The values of D, L, W, etc. given by ASCE 7-98 are nominal loads (not maximum or ultimate)

- During its design life, a structure can be subjected to some maximum or ultimate loads caused by combinations of D, L, or W loading.

- The ultimate load on the structure can be calculated using <u>factored load combinations</u>, which are given by ASCE and AISC (see pages 2-10 and 2-11 of AISC manual). The most relevant of these load combinations are given below:

$$1.4\,D \tag{4.2 – 1}$$
$$1.2\,D + 1.6\,L + 0.5\,(L_r \text{ or } S) \tag{4.2 – 2}$$
$$1.2\,D + 1.6\,(L_r \text{ or } S) + (0.5\,L \text{ or } 0.8\,W) \tag{4.2 – 3}$$
$$1.2\,D + 1.6\,W + 0.5\,L + 0.5\,(L_r \text{ or } S) \tag{4.2 – 4}$$
$$0.9\,D + 1.6\,W \tag{4.2 – 5}$$

<u>Step II. Conduct linear elastic structural analysis</u>

- Determine the design forces ($P_u$, $V_u$, and $M_u$) for each structural member

<u>Step III. Design the members</u>

- The failure (design) strength of the designed member must be greater than the corresponding design forces calculated in Step II. See Equation (4.3) below:

$$\phi\,R_n > \sum \gamma_i\,Q_i \tag{4.3}$$

- Where, $R_n$ is the calculated failure strength of the member

- $\phi$ is the resistance factor used to account for the reliability of the material behavior and equations for $R_n$

- $Q_i$ is the nominal load

- $\gamma_i$ is the load factor used to account for the variability in loading and to estimate the ultimate loading condition.

2.3.2  Design Strength of Tension Members

- Yielding of the gross section will occur when the stress $f$ reaches $F_y$.

$$f = \frac{P}{A_g} = F_y$$

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Therefore, nominal yield strength = $P_n = A_g F_y$                   (4.4)

Factored yield strength = $\phi_t P_n$                 (4.5)

where, $\phi_t = 0.9$ for tension yielding limit state

- See the AISC manual, section on <u>specifications</u>, Chapter D (page 16.1 –24)

- Facture of the net section will occur after the stress on the net section area reaches the ultimate stress $F_u$

$$f = \frac{P}{A_e} = F_u$$

Therefore, nominal fracture strength = $P_n = A_e F_u$

Where, $A_e$ is the effective net area, which may be equal to the net area or smaller.

The topic of $A_e$ will be addressed later.

Factored fracture strength = $\phi_t A_e F_u$            (4.6)

Where, $\phi_t = 0.75$ for tension fracture limit state   (See page 16.1-24 of AISC manual)


### 2.3.3 Important notes

- <u>Note 1</u>. Why is fracture (& not yielding) the relevant limit state at the net section?

  Yielding will occur first in the net section. However, the deformations induced by yielding will be localized around the net section. These localized deformations will *not* cause excessive deformations in the complete tension member. Hence, yielding at the net section will *not* be a failure limit state.

- <u>Note 2.</u> Why is the resistance factor ($\phi_t$) smaller for fracture than for yielding?

  The smaller resistance factor for fracture ($\phi_t = 0.75$ as compared to $\phi_t = 0.90$ for yielding) reflects the more serious nature and consequences of reaching the fracture limit state.

- <u>Note 3.</u> What is the design strength of the tension member?

The design strength of the tension member will be the <u>lesser</u> value of the strength for the two limit states (gross section yielding and net section fracture).

- <u>Note 4.</u> Where are the $F_y$ and $F_u$ values for different steel materials?

  The yield and ultimate stress values for different steel materials are noted in Table 2 in the *AISC* manual on pages *16.1–141* and *16.1–142*.

- <u>Note 5.</u> What are the most common steels for structural members?

  See Table 2-1 in the *AISC* manual on pages *2–24* and *2-25*. According to this Table: the preferred material for *W* shapes is *A992* ($F_y$ = 50 ksi; $F_u$ = 65 ksi); the preferred material for *C, L , M* and *S* shapes is *A36* ($F_y$ = 36 ksi; $F_u$ = 58 ksi). All these shapes are also available in *A572 Gr. 50* ($F_y$ = 50 ksi; $F_u$ = 65 ksi).

- <u>Note 6.</u> What is the amount of area to be deducted from the gross area to account for the presence of bolt-holes?

  - The *nominal* diameter of the hole ($d_h$) is equal to the bolt diameter ($d_b$) + 1/16 in.

  - However, the bolt-hole fabrication process damages additional material around the hole diameter.

  - Assume that the material damage extends 1/16 *in.* around the hole diameter.

  - Therefore, for calculating the net section area, assume that the gross area is *reduced by a hole diameter* equal to the nominal hole-diameter + 1/16 in.

Example 3.1 A 5 x ½ bar of A572 Gr. 50 steel is used as a tension member. It is connected to a gusset plate with six 7/8 in. diameter bolts as shown in below. Assume that the effective net area $A_e$ equals the actual net area $A_n$ and compute the tensile design strength of the member.
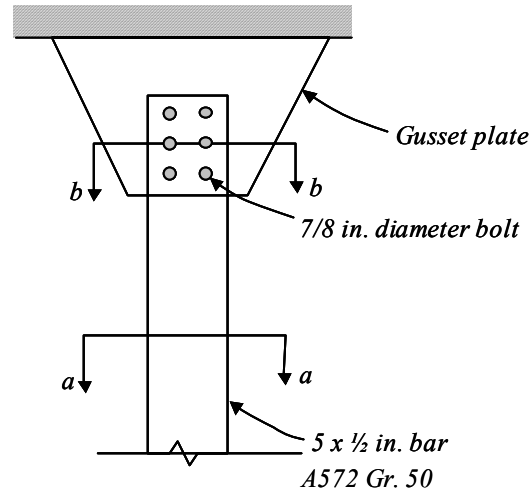


Solution

- Gross section area = $A_g$ = 5 x ½ = 2.5 in$^2$

- Net section area ($A_n$)

    - Bolt diameter = $d_b$ = 7/8 in.

    - Nominal hole diameter = $d_h$ = 7/8 + 1/16 in. = 15/16 in.

    - Hole diameter for calculating net area = 15/16 + 1/16 in. = 1 in.

    - Net section area = $A_n$ = (5 – 2 x (1)) x ½ = 1.5 in$^2$

- Gross yielding design strength = $\phi_t P_n = \phi_t F_y A_g$

    - Gross yielding design strength = 0.9 x 50 ksi x 2.5 in$^2$ = 112.5 kips

- Fracture design strength = $\phi_t P_n = \phi_t F_u A_e$

    - Assume $A_e = A_n$ (only for this problem)

    - Fracture design strength = 0.75 x 65 ksi x 1.5 in$^2$ = 73.125 kips

- Design strength of the member in tension = smaller of 73.125 kips and 112.5 kips

- Therefore, design strength = 73.125 kips (*net section fracture controls*).

Example 3.2 A single angle tension member, *L* 4 x 4 x 3/8 in. made from A36 steel is connected to a gusset plate with 5/8 in. diameter bolts, as shown in Figure below. The service loads are 35 kips dead load and 15 kips live load. Determine the adequacy of this member using AISC specification. Assume that the effective net area is 85% of the computed net area. (*Calculating the effective net area will be taught in the next section*).

- Gross area of angle = $A_g$ = 2.86 in$^2$       (*from Table 1-7 on page 1-36 of AISC*)



*L* 4 x 4 x 3/8

$d_b$ = 5/8 in.

*Section a-a*

*Gusset plate*
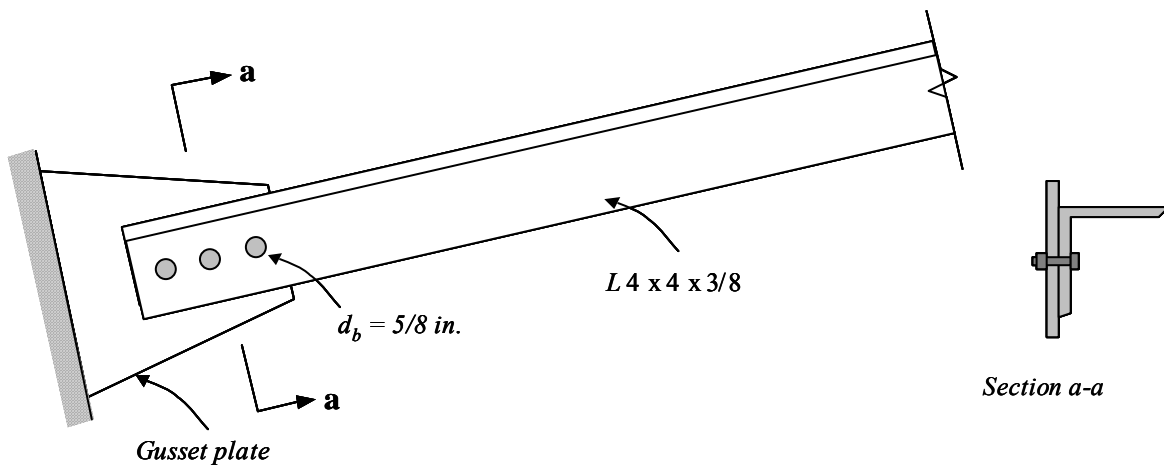
- Net section area = $A_n$

  - Bolt diameter = 5/8 in.

  - Nominal hole diameter = 5/8 + 1/16 = 11/16 in.

  - Hole diameter for calculating net area = 11/16 + 1/16 = 3/4 in.

  - Net section area = $A_g$ − (3/4) x 3/8 = 2.86 − 3/4 x 3/8 = 2.579 in$^2$

- Effective net area = $A_e$ = 0.85 x 2.579 in$^2$ = 2.192 in$^2$

- Gross yielding design strength = $\phi_t\, A_g\, F_y$ = 0.9 x 2.86 in$^2$ x 36 ksi = 92.664 kips

- Net section fracture = $\phi_t\, A_e\, F_u$ = 0.75 x 2.192 in$^2$ x 58 ksi = 95.352 kips

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- Design strength = 92.664 kips    (gross yielding governs)

- Ultimate (design) load acting for the tension member = $P_u$

  - The ultimate (design) load can be calculated using factored load combinations given on page *2-11* of the *AISC* manual, or Equations (*4.2-1 to 4.2-5*) of notes (see pg. 4)

  - According to these equations, two loading combinations are important for this problem. These are: (1) 1.4 *D*; and (2) 1.2 *D* + 1.6 *L*

  - The corresponding ultimate (design) loads are:

    1.4 x ($P_D$) = 1.4 (35) = 49 kips

    1.2 ($P_D$) + 1.6 ($P_L$) = 66 kips                    (controls)

  - The ultimate design load for the member is 66 kips, where the *factored* dead + live loading condition controls.

- Compare the design strength with the ultimate design load

  - The design strength of the member (92.664 kips) is greater than the ultimate design load (66 kips).

  - $\phi_t P_n$ (92.664 kips) > $P_u$ (66 kips)

- The *L* 4 x 4 x 3/8 in. made from A36 steel is adequate for carrying the factored loads.


2.4  EFFECTIVE  NET  AREA

- The *connection* has a significant influence on the performance of a tension member. A connection almost always weakens the member, and a measure of its influence is called joint efficiency.

- Joint efficiency is a function of: (a) material ductility; (b) fastener spacing; (c) stress concentration at holes; (d) fabrication procedure; and (e) **shear lag**.

- All factors contribute to reducing the effectiveness but <u>shear lag is the most important</u>.

- Shear lag occurs when the tension force is not transferred *simultaneously* to all elements of the cross-section. This will occur when some elements of the cross-section are not connected.

- For example, see Figure 4.3 below, where only one leg of an angle is bolted to the gusset plate.
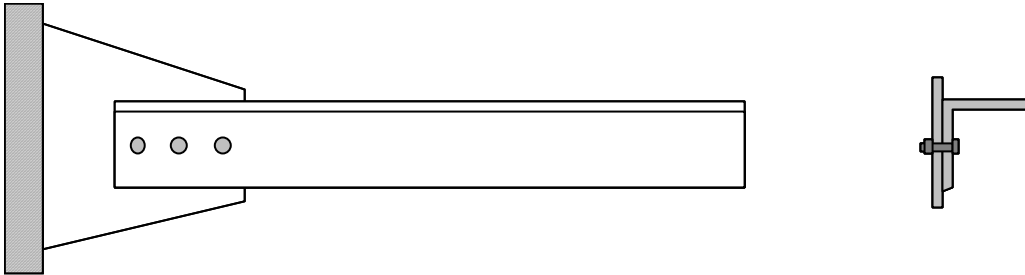


**Figure 4.3** Single angle with bolted connection to only one leg.

- A consequence of this partial connection is that the connected element becomes overloaded and the unconnected part is not fully stressed.

- Lengthening the connection region will reduce this effect

- Research indicates that shear lag can be accounted for by using a reduced or effective net area $A_e$

- Shear lag affects both bolted and welded connections. Therefore, the effective net area concept applied to both types of connections.

  - For bolted connection, the effective net area is $A_e = U A_n$

  - For welded connection, the effective net area is $A_e = U A_g$

- Where, the reduction factor *U* is given by:

$$U = 1 - \frac{\overline{x}}{L} \leq 0.9 \qquad (4.7)$$

- Where, $\overline{x}$ is the distance from the centroid of the connected area to the plane of the connection, and L is the length of the connection.

    - If the member has two symmetrically located planes of connection, $\overline{x}$ is measured from the centroid of the nearest one – half of the area.

    - Additional approaches for calculating $\overline{x}$ for different connection types are shown in the *AISC* manual on page **16.1-178.**

- The distance L is defined as the length of the connection in the direction of load.

    - For bolted connections, L is measured from the center of the bolt at one end to the center of the bolt at the other end.

    - For welded connections, it is measured from one end of the connection to other.

    - If there are weld segments of different length in the direction of load, L is the length of the longest segment.

    - Example pictures for calculating L are given on page **16.1-179** of *AISC.*

- The AISC manual also gives values of *U* that can be used instead of calculating $\overline{x}/L$.

    - They are based on average values of $\overline{x}/L$ for various <u>bolted</u> connections.

    - For W, M, and S shapes with width-to-depth ratio of at least 2/3 and for Tee shapes cut from them, if the connection is through the flanges with at least three fasteners per line in the direction of applied load …………………………………………………...*U = 0.90*

    - For all other shapes with at least three fasteners per line …………………... *U = 0.85*

    - For all members with only two fasteners per line …………………………… *U = 0.75*

    - For better idea, see Figure 3.8 on page 41 of the <u>Segui text-book</u>.

    - These values are acceptable but not the best estimate of U

    - If used in the <u>exam or homeworks</u>, full points for calculating U will not be given

Example 3.3 Determine the effective net area and the corresponding design strength for the single angle tension member of *Example 3.2*. The tension member is an *L* 4 x 4 x 3/8 in. made from A36 steel. It is connected to a gusset plate with 5/8 in. diameter bolts, as shown in Figure below. The spacing between the bolts is 3 in. center-to-center.

- Compare your results with those obtained for *Example 3.2*.
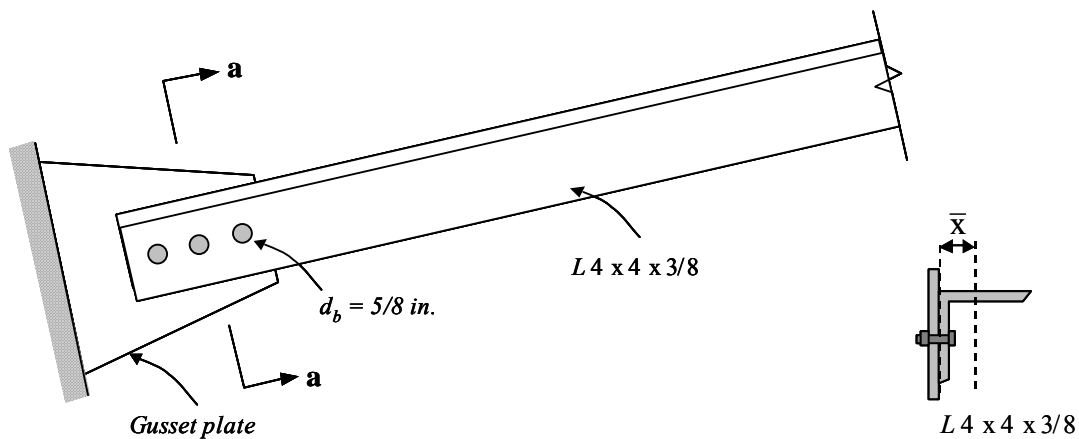


- Gross area of angle = $A_g$ = 2.86 in$^2$     (*from Table 1-7 on page 1-36 of AISC*)

- Net section area = $A_n$

  - Bolt diameter = 5/8 in.

  - Hole diameter for calculating net area = 11/16 + 1/16 = 3/4 in.

  - Net section area = $A_g$ – (3/4) x 3/8 = 2.86 – 3/4 x 3/8 = 2.579 in$^2$

- $\bar{x}$ is the distance from the centroid of the area connected to the plane of connection

  - For this case $\bar{x}$ is equal to the distance of centroid of the angle from the edge.

  - This value is given in the *Table 1-7 on page 1-36 of* the *AISC* manual.

  - $\bar{x}$ = 1.13 in.

- L is the length of the connection, which for this case will be equal to 2 x 3.0 in.

- L = 6.0 in.

- $U = 1 - \dfrac{\overline{x}}{L} = 1 - \dfrac{1.13}{6.0} = 0.8116$ in.

- Effective net area = $A_e$ = 0.8116 x 2.579 in$^2$ = 2.093 in$^2$

- Gross yielding design strength = $\phi_t A_g F_y$ = 0.9 x 2.86 in$^2$ x 36 ksi = 92.664 kips

- Net section fracture = $\phi_t A_e F_u$ = 0.75 x 2.093 in$^2$ x 58 ksi = 91.045 kips

- <u>Design strength = 91.045 kips     (net section fracture governs)</u>

- In Example 3.2

  - Factored load = $P_u$ = 66.0 kips

  - Design strength = $\phi_t P_n$ = 92.66 kips  (*gross section yielding governs*)

  - Net section fracture strength = $\phi_t P_n$ = 95.352 kips  (*assuming* $A_e$ = 0.85)

- Comparing Examples 3.2 and 3.3

  - Calculated value of $U$ (0.8166) is less than the assumed value (0.85)

  - The assumed value was <u>unconservative</u>.

  - It is preferred that the $U$ value be specifically calculated for the section.

  - After including the calculated value of $U$, net section fracture governs the design strength, but the member is still adequate from a design standpoint.

Example 3.4 Determine the design strength of an ASTM A992 W8 x 24 with four lines if ¾ in.

diameter bolts in standard holes, two per flange, as shown in the Figure below.

Assume the holes are located at the member end and the connection length is 9.0 in. Also

calculate at what length this tension member would cease to satisfy the slenderness limitation in

LRFD specification B7

¾ in. diameter bolts

W 8 x 24

3 in.      3 in.      3 in.

Holes in beam flange
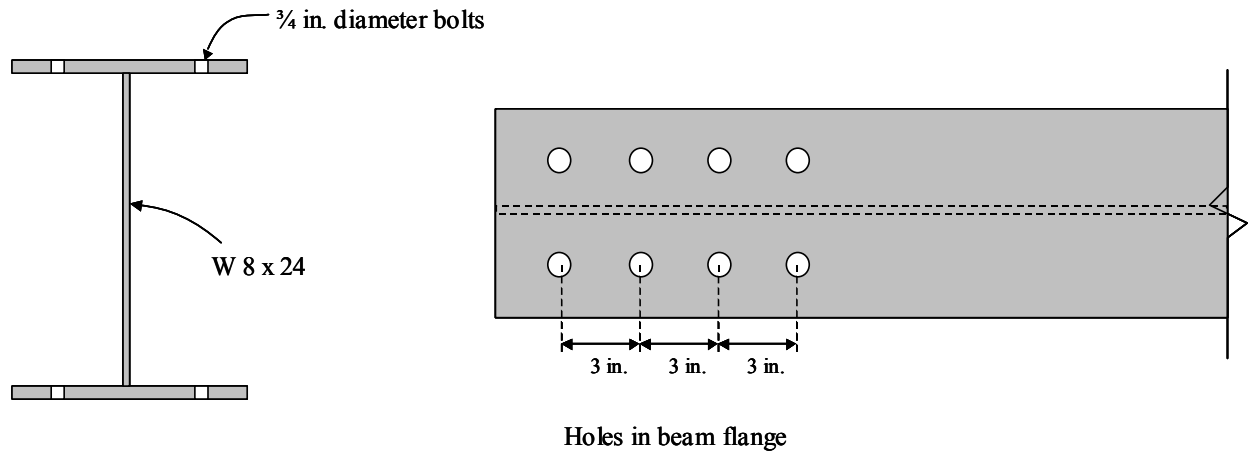
Solution:

- For ASTM A992 material: $F_y$ = 50 ksi; and $F_u$ = 65 ksi

- For the W8 x 24 section:

  - $A_g$ = 7.08 in$^2$         d = 7.93 in.
  - $t_w$ = 0.285 in.          $b_f$ = 6.5 in.
  - $t_f$ = 0.4 in.            $r_y$ = 1.61 in.

- Gross yielding design strength = $\phi_t P_n = \phi_t A_g F_y$ = 0.90 x 7.08 in$^2$ x 50 ksi = <u>319 kips</u>

- Net section fracture strength = $\phi_t P_n = \phi_t A_e F_u$ = 0.75 x $A_e$ x 65 ksi

  - $A_e = U A_n$              - for bolted connection

  - $A_n = A_g$ – (no. of holes) x (diameter of hole) x (thickness of flange)

    $A_n$ = 7.08 – 4 x (diameter of bolt + 1/8 in.) x 0.4 in.

    <u>$A_n$ = 5.68 in$^2$</u>

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- $U = 1 - \dfrac{\overline{x}}{L} \leq 0.90$

- What is $\overline{x}$ for this situation?

  *$\overline{x}$ is the distance from the edge of the flange to the centroid of the half (T) section*

$$\overline{x} = \frac{(b_f \times t_f) \times \dfrac{t_f}{2} + (\dfrac{d - 2\,t_f}{2} \times t_w) \times (\dfrac{d + 2\,t_f}{4})}{b_f \times t_f + \dfrac{d}{2} \times t_w} = \frac{6.5 \times 0.4 \times 0.2 + 3.565 \times 0.285 \times 2.1825}{6.5 \times 0.4 + 3.565 \times 0.285} = 0.76$$

- $\overline{x}$ can be obtained from the <u>dimension</u> tables for Tee section *WT 4 x 12*. See page *1-50*

  and *1-51* of the AISC manual:

  <u>$\overline{x}$ = 0.695 in.</u>

- The calculated value is *not accurate* due to the deviations in the geometry

- $U = 1 - \dfrac{\overline{x}}{L} = 1 - \dfrac{0.695}{9.0} = 0.923$

- But, $U \leq 0.90$. Therefore, assume <u>$U = 0.90$</u>

- Net section fracture strength = $\phi_t\, A_e\, F_u$ = 0.75 x 0.9 x 5.68 x 65 = <u>249.2 kips</u>

- The design strength of the member is controlled by net section fracture = <u>249.2 kips</u>

- According to LRFD specification B7, the maximum unsupported length of the member is

  limited to 300 $r_y$ = 300 x 1.61 in. = 543 in. = <u>40.3 ft.</u>

2.4.1 Special cases for welded connections

- If some elements of the cross-section are not connected, then $A_e$ will be less than $A_n$

  - For a rectangular bar or plate $A_e$ will be equal to $A_n$

  - However, if the connection is by longitudinal welds at the ends as shown in the figure

    below, then $A_e = UA_g$

    Where,         $U = 1.0$                    for $L \geq w$
    $U = 0.87$                  for $1.5 \, w \leq L < 2 \, w$
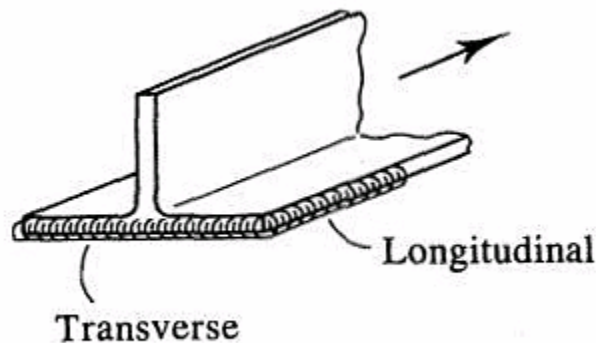    $U = 0.75$                  for $w \leq L < 1.5 \, w$
    $L$ = length of the pair of welds $\geq w$
    $w$ = distance between the welds or width of plate/bar
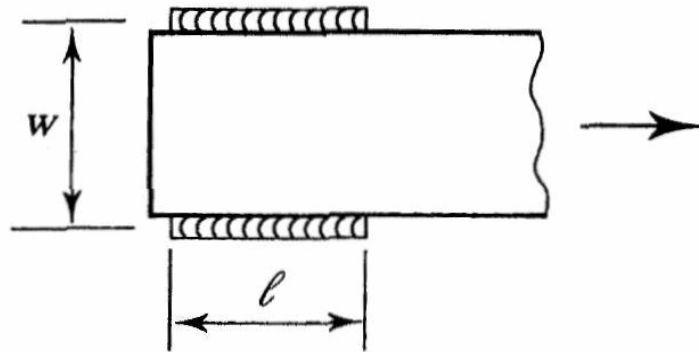


- AISC Specification B3 gives another special case for welded connections.

  For any member connected by *transverse welds alone,*

     $A_e$ = area of the connected element of the cross-section



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Example 3.5 Consider the welded single angle $L$ 6x 6 x ½ tension member made from A36 steel shown below. Calculate the tension design strength.



$L6 \times 6 \times ½$
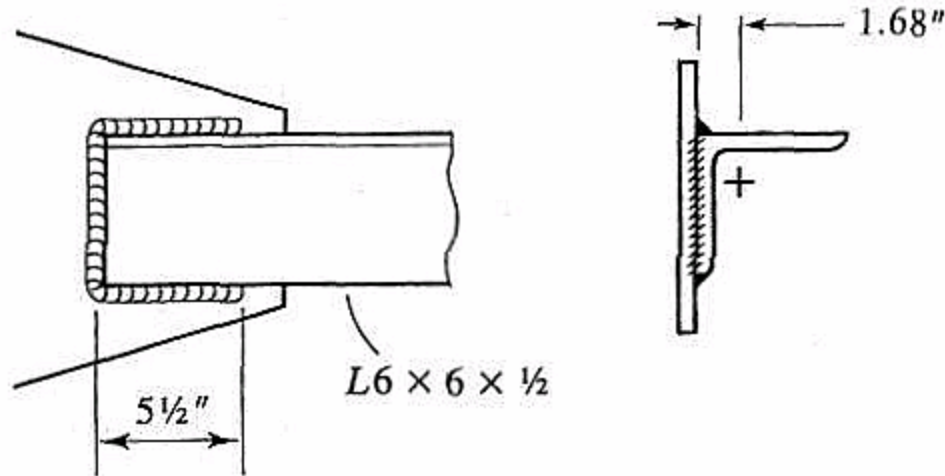
Solution

- $A_g = 5.00$ in$^2$

- $A_n = 5.00$ in$^2$     - because it is a welded connection

- $A_e = U\,A_n$     - where, $U = 1 - \dfrac{\overline{x}}{L}$

   - $\overline{x} = 1.68$ in. for this welded connection

   - $L = 6.0$ in. for this welded connection

   - $U = 1 - \dfrac{1.168}{6.0} = 0.72$

- Gross yielding design strength = $\phi_t\,F_y\,A_g = 0.9 \times 36 \times 5.00 = 162$ kips

- Net section fracture strength = $\phi_t\,F_u\,A_e = 0.75 \times 58 \times 0.72 \times 5.00 = 156.6$ kips

- Design strength = 156.6 kips  (*net section fracture governs*)

(a)



(b)



(c)
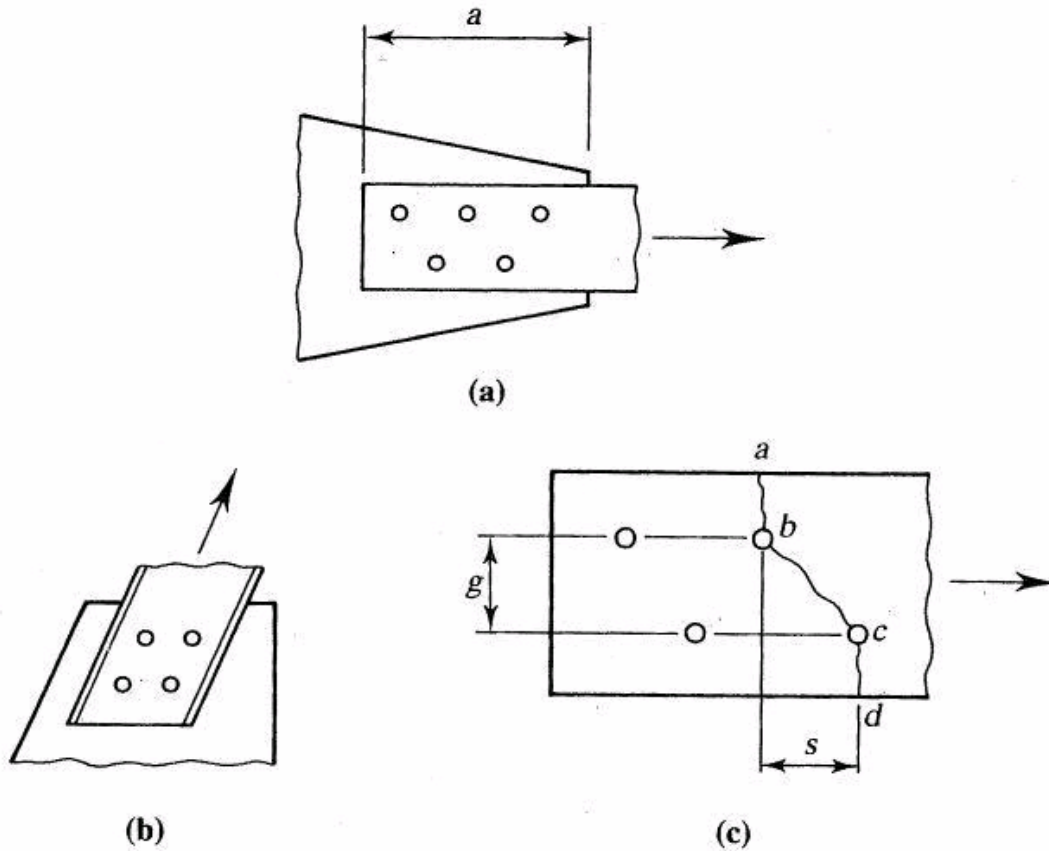
## 2.5 STAGGERED BOLTS

For a bolted tension member, the connecting bolts can be staggered for several reasons:

(1) To get more capacity by increasing the effective net area

(2) To achieve a smaller connection length

(3) To fit the geometry of the tension connection itself.


- For a tension member with staggered bolt holes (see example figure above), the relationship $f$ = P/A does not apply and the stresses are a combination of tensile and shearing stresses on the inclined portion $b$-$c$.

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- Net section fracture can occur along any zig-zag or straight line. For example, fracture can occur along the inclined path *a-b-c-d* in the figure above. <u>However, all possibilities must be examined</u>.

- Empirical methods have been developed to calculate the net section fracture strength According to AISC Specification B2

  - net width = gross width $- \sum d + \sum \dfrac{s^2}{4\,g}$

  - where, d is the diameter of hole to be deducted ($d_h + 1/16$, or $d_b + 1/8$)

  - $s^2/4g$ is added for each gage space in the chain being considered

  - *s* is the longitudinal spacing (pitch) of the bolt holes in the direction of loading

  - *g* is the transverse spacing (gage) of the bolt holes perpendicular to loading dir.

  - net area ($A_n$) = net width x plate thickness

  - effective net area ($A_e$) = $U\,A_n$      where U = 1- $\bar{x}$ /L

  - net fracture design strength = $\phi_t\,A_e\,F_u$      ($\phi_t$ = 0.75)

**EXAMPLE 3.6** Compute the smallest net area for the plate shown below: The holes are for 1 in. diameter bolts.



- The effective hole diameter is $1 + 1/8 = 1.125$ in.

- For line *a-b-d-e*

  $w_n = 16.0 - 2 (1.125) = 13.75$ in.

- For line *a-b-c-d-e*

  $w_n = 16.0 - 3 (1.125) + 2 \times 3^2/ (4 \times 5) = 13.52$ in.

- The line *a-b-c-d-e* governs:

- $A_n = t\, w_n = 0.75 (13.52) = 10.14$ in$^2$

Note

- Each fastener resists an equal share of the load

- Therefore different potential failure lines may be subjected to different loads.

- For example, line *a-b-c-d-e* must resist the full load, whereas *i-j-f-h* will be subjected to 8/11 of the applied load. The reason is that 3/11 of the load is transferred from the member before *i-j-f-h* received any load.

- **Staggered bolts in angles.** If staggered lines of bolts are present in both legs of an angle, then the net area is found by first unfolding the angle to obtain an equivalent plate. This plate is then analyzed like shown above.

  - The unfolding is done at the middle surface to obtain a plate with gross width equal to the sum of the leg lengths minus the angle thickness.

  - AISC Specification B2 says that any gage line crossing the heel of the angle should be reduced by an amount equal to the angle thickness.

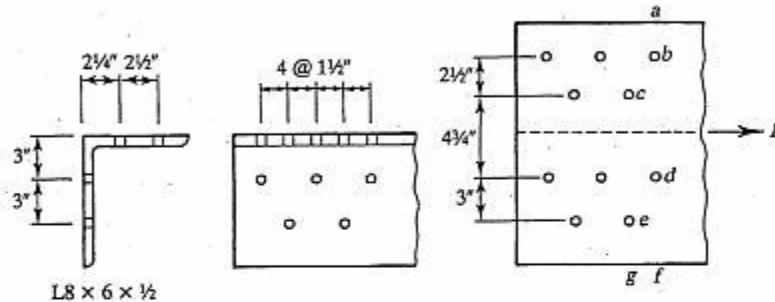  - See Figure below. For this situation, the distance g will be $= 3 + 2 - \frac{1}{2}$ in.



L5 × 5 × ½
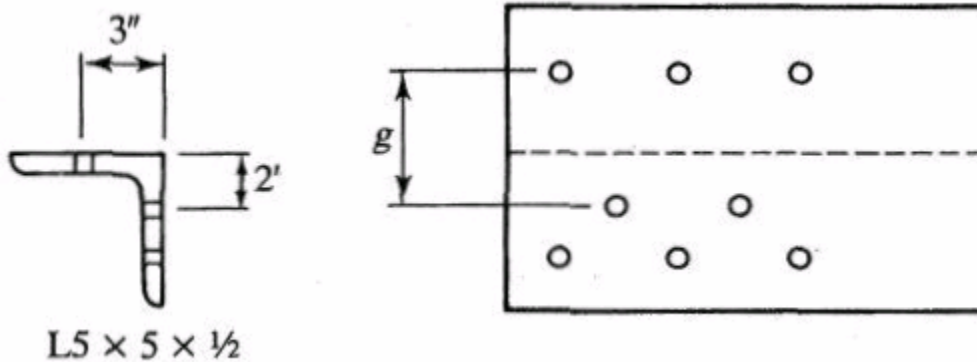
## EXAMPLE 3.6

Find the design tensile strength of the angle shown in Figure 3.16. A36 steel is used, and holes are for ⅞-inch-diameter bolts.

**SOLUTION**
Compute the net width:

$$w_g = 8 + 6 - \tfrac{1}{2} = 13.5 \text{ in.}$$

### FIGURE 3.16



L8 × 6 × ½

Effective hole diameter $= \tfrac{7}{8} + \tfrac{1}{8} = 1$ in.

For line *abdf*,

$$w_n = 13.5 - 2(1) = 11.5 \text{ in.}$$

For line *abceg*,

$$w_n = 13.5 - 3(1) + \frac{(1.5)^2}{4(2.5)} = 10.73 \text{ in.}$$

Because ¹⁄₁₀ of the load has been transferred from the member by the fastener at *d*, this potential failure line must resist only ⁹⁄₁₀ of the load. Therefore the net width of 10.73 inch should be multiplied by ¹⁰⁄₉ to obtain a net width that can be compared with those lines that resist the full load. Use $w_n = 10.73(^{10}\!/\!_9) = 11.92$ inch. For line *abcdeg*,

$$g_{cd} = 3 + 2.25 - 0.5 = 4.75 \text{ in.}$$

$$w_n = 13.5 - 4(1) + \frac{(1.5)^2}{4(2.5)} + \frac{(1.5)^2}{4(4.75)} + \frac{(1.5)^2}{4(3)} = 10.03 \text{ in.}$$

The last case controls:

$$A_n = t(w_n) = 0.5(10.03) = 5.015 \text{ in.}^2$$

Both legs of the angle are connected, so

$$A_e = A_n = 5.015 \text{ in.}^2$$

The design strength based on fracture is

$$\phi_t P_n = 0.75 F_u A_e = 0.75(58)(5.015) = 218 \text{ kips}$$

The design strength based on yielding is

$$\phi_t P_n = 0.90 F_y A_g = 0.90(36)(6.75) = 219 \text{ kips}$$

**ANSWER**
Fracture controls; design strength = 218 kips.

## 2.6 BLOCK SHEAR

- For some connection configurations, the tension member can fail due to 'tear-out' of material at the connected end. This is called _block shear_.

- For example, the single angle tension member connected as shown in the Figure below is susceptible to the phenomenon of _block shear_.
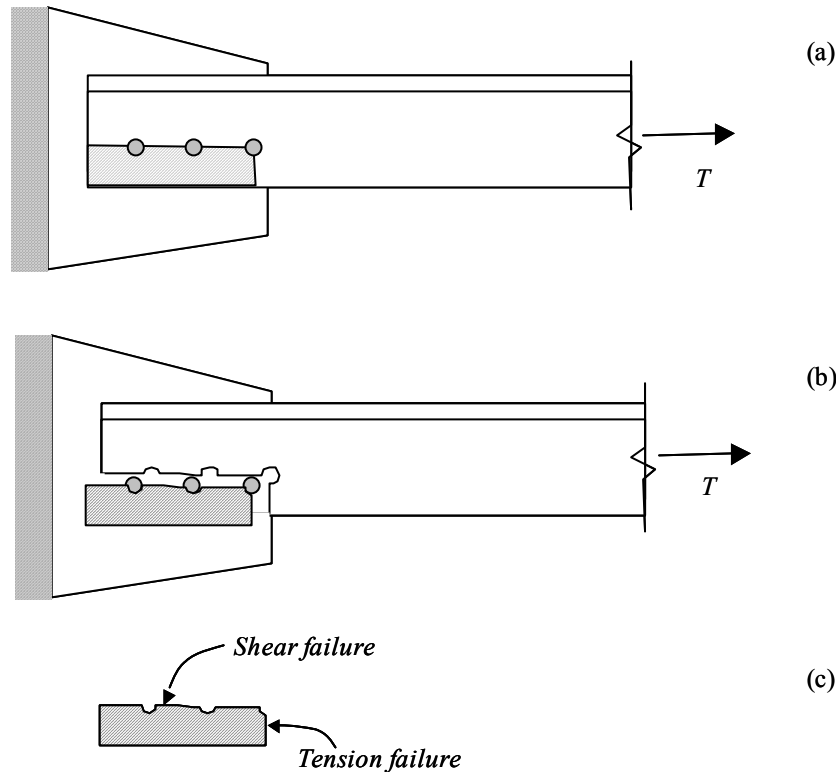


**Figure 4.4** Block shear failure of single angle tension member

- For the case shown above, shear failure will occur along the longitudinal section a-b and tension failure will occur along the transverse section b-c

- AISC Specification (SPEC) Chapter D on tension members does not cover block shear failure explicitly. But, it directs the engineer to the Specification Section J4.3

- Block shear strength is determined as the <u>sum</u> of the shear strength on a failure path and the tensile strength on a perpendicular segment.

  - Block shear strength = net section fracture strength on shear path + gross yielding strength on the tension path

  - **<u>OR</u>**

  - Block shear strength = gross yielding strength of the shear path + net section fracture strength of the tension path

- Which of the two calculations above governs?

  - See page **16.1 – 67** (Section J4.3) of the AISC manual

  - When $F_u A_{nt} \geq 0.6 F_u A_{nv}$; $\phi_t R_n = \phi (0.6 F_y A_{gv} + F_u A_{nt}) \leq \phi (0.6 F_u A_{nv} + F_u A_{nt})$

  - When $F_u A_{nt} < 0.6 F_u A_{nv}$; $\phi_t R_n = \phi (0.6 F_u A_{nv} + F_y A_{gt}) \leq \phi (0.6 F_u A_{nv} + F_u A_{nt})$

  - Where, $\phi = 0.75$

    $A_{gv}$ = gross area subject to shear

    $A_{gt}$ = gross area subject to tension

    $A_{nv}$ = net area subject to shear

    $A_{nt}$ = net area subject to tension

**EXAMPLE 3.8** Calculate the block shear strength of the single angle tension member considered in Examples 3.2 and 3.3. The single angle *L 4 x 4 x 3/8* made from A36 steel is connected to the gusset plate with 5/8 in. diameter bolts as shown below. The bolt spacing is 3 in. center-to-center and the edge distances are 1.5 in and 2.0 in as shown in the Figure below.

Compare your results with those obtained in *Example 3.2 and 3.3*
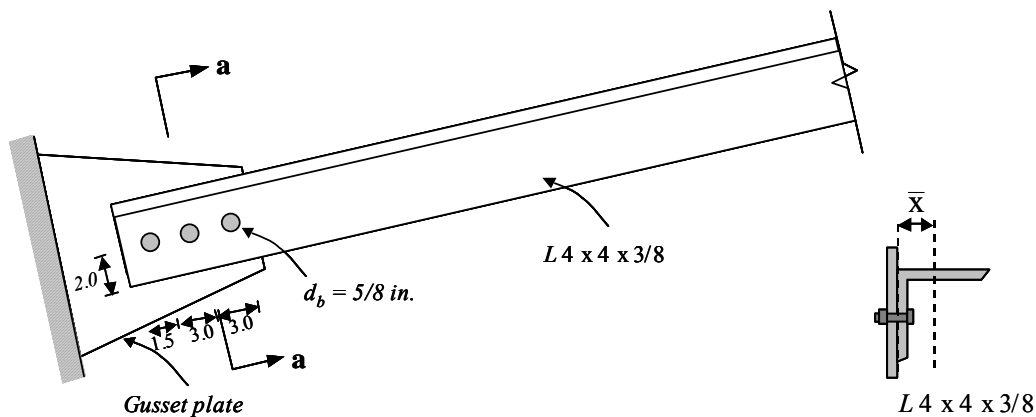


- Step I. Assume a block shear path and calculate the required areas
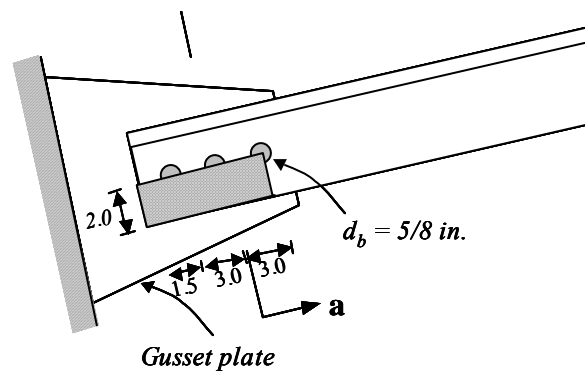


- $A_{gt}$ = gross tension area = 2.0 x 3/8 = 0.75 in$^2$

- $A_{nt}$ = net tension area = 0.75 – **0.5** x (5/8 + 1/8) x 3/8 = 0.609 in$^2$

- $A_{gv}$ = gross shear area = (3.0 + 3.0 +1.5) x 3/8 = 2.813 in$^2$

- $A_{nv}$ = net tension area = 2.813 – **2.5** x (5/8 + 1/8) x 3/8 = 2.109 in$^2$

- Step II. Calculate which equation governs

  - $0.6\, F_u\, A_{nv} = 0.6 \times 58 \times 2.109 = 73.393$ kips

  - $F_u\, A_{nt} = 58 \times 0.609 = 35.322$ kips

  - $0.6\, F_u\, A_{nv} > F_u\, A_{nt}$

  - Therefore, equation with fracture of shear path governs

- Step III. Calculate block shear strength

  - $\phi_t\, R_n = 0.75\, (0.6\, F_u\, A_{nv} + F_y\, A_{gt})$

  - $\phi_t\, R_n = 0.75\, (73.393 + 36 \times 0.75) = \mathbf{75.294\ kips}$

- Compare with results from previous examples

  Example 3.2:

   Ultimate factored load $= P_u = 66$ kips

   Gross yielding design strength $= \phi_t\, P_n = 92.664$ kips

   Assume $A_e = 0.85\, A_n$

   Net section fracture strength $= 95.352$ kips

   Design strength $= 92.664$ kips (gross yielding governs)


  Example 3.3

   Calculate $A_e = 0.8166\, A_n$

   Net section fracture strength $= 91.045$ kips

   Design strength $= 91.045$ kips (net section fracture governs)

   Member is still adequate to carry the factored load $(P_u) = 66$ kips


  Example 3.8

   Block shear fracture strength $= 75.294$ kips

   Design strength $= 75.294$ kips (block shear fracture governs)

   Member is still adequate to carry the factored load $(P_u) = 66$ kips

- *Bottom line*:

  - Any of the three limit states (gross yielding, net section fracture, or block shear failure) can govern.
  - The design strength for all three limit states has to be calculated.
  - The member design strength will be the smallest of the three calculated values
  - The member design strength must be greater than the ultimate factored design load in tension.

**Practice Example** Determine the <u>design</u> tension strength for a single channel C15 x 50 connected to a 0.5 in. thick gusset plate as shown in Figure. Assume that the holes are for 3/4 in. diameter bolts and that the plate is made from structural steel with yield stress ($F_y$) equal to 50 ksi and ultimate stress ($F_u$) equal to 65 ksi.
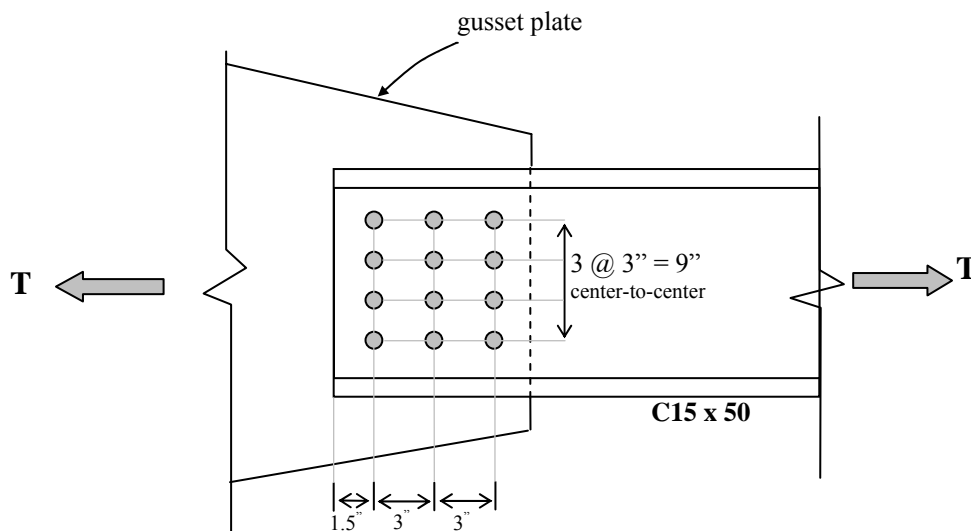


- **Limit state of yielding due to tension:**
$$\phi T_n = 0.9 * 50 * 14.7 = 662 kips$$
- **Limit state of fracture due to tension:**
$$A_n = A_g - nd_e t = 14.7 - 4\left(\frac{7}{8}\right)(0.716) = 12.19 in^2$$
$$A_e = UA_n = \left(1 - \frac{x}{L}\right)A_n = \left(1 - \frac{0.798}{6}\right) * 12.19 = 10.57 in^2$$

**Check**: $U = 0.867 \le 0.9$ OK.

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**Note**: The connection eccentricity, x, for a C15X50 can be found on page 1-51 (LRFD).

$$\phi T_n = 0.75 * 65 * 10.57 = 515 kips$$

▪ **Limit state of block shear rupture:**

$$0.6 F_u A_{nv} = 0.6 * 65 * \left[ 2 * \left( 7.5 - 2.5 * \frac{7}{8} \right) \right] * 0.716 = 296.6925$$

$$F_u A_{nt} = 65 * \left[ 9 - 3 \left( \frac{7}{8} \right) \right] * 0.716 = 296.6925$$

$$F_u A_{nt} \geq 0.6 F_u A_{nv}$$

$$\therefore \phi R_n = \phi \left[ 0.6 F_y A_{gv} + F_u A_{nt} \right] = 0.75 \left[ 0.6 * 50 * 15 * 0.716 + 65 * \frac{296.6925}{65} \right] = 464 kips$$

Block shear rupture is the critical limit state and the design tension strength is 464kips.

## 2.7 Design of tension members

- The design of a tension member involves <u>finding the lightest steel section</u> (angle, wide-flange, or channel section) with design strength ($\phi P_n$) greater than or equal to the maximum factored design tension load ($P_u$) acting on it.

  - $\phi P_n \geq P_u$

  - $P_u$ is determined by structural analysis for factored load combinations

  - $\phi P_n$ is the design strength based on the *gross section yielding*, *net section fracture*, and *block shear rupture* limit states.

- For gross yielding limit state, $\phi P_n = 0.9 \times A_g \times F_y$

  - Therefore, $0.9 \times A_g \times F_y \geq P_u$
  - Therefore, $A_g \geq \dfrac{P_u}{0.9 \times F_y}$

- For net section fracture limit state, $\phi P_n = 0.75 \times A_e \times F_u$

  - Therefore, $0.75 \times A_e \times F_u \geq P_u$
  - Therefore, $A_e \geq \dfrac{P_u}{0.75 \times F_u}$
  - But, $A_e = U A_n$

  - Where, $U$ and $A_n$ depend on the end connection.

- Thus, designing the tension member goes hand-in-hand with designing the end connection, which we have not covered so far.

- Therefore, for this chapter of the course, the end connection details will be given in the examples and problems.

- The AISC manual tabulates the tension design strength of standard steel sections

  - Include: wide flange shapes, angles, tee sections, and double angle sections.

- The gross yielding design strength and the net section fracture strength of each section is tabulated.

- This provides a great *starting point* for selecting a section.

- **There is one serious limitation**

  - The net section fracture strength is tabulated for an <u>assumed value of $U = 0.75$</u>, obviously because the precise connection details are not known

  - For all W, Tee, angle and double-angle sections, $A_e$ is assumed to be $= 0.75\ A_g$

  - The engineer can **<u>first</u>** select the tension member based on the tabulated gross yielding and net section fracture strengths, and then <u>check the net section fracture strength and the block shear strength using the actual connection details</u>.

- Additionally for each shape the manual tells the value of $A_e$ below which net section fracture will control:

  - Thus, for W shapes net section fracture will control if $A_e < 0.923\ A_g$

  - For single angles, net section fracture will control if $A_e < 0.745\ A_g$

  - For Tee shapes, net section fracture will control if $A_e < 0.923$

  - For double angle shapes, net section fracture will control if $A_e < 0.745\ A_g$

- **Slenderness limits**

  - Tension member slenderness l/r must preferably be limited to 300 as per LRFD specification B7

**Example 3.10** Design a member to carry a factored maximum tension load of 100 kips.

(a) Assume that the member is a wide flange connected through the flanges using eight ¾ in. diameter bolts in two rows of four each as shown in the figure below. The center-to-center distance of the bolts in the direction of loading is 4 in. The edge distances are 1.5 in. and 2.0 in. as shown in the figure below. Steel material is A992



Holes in beam flange
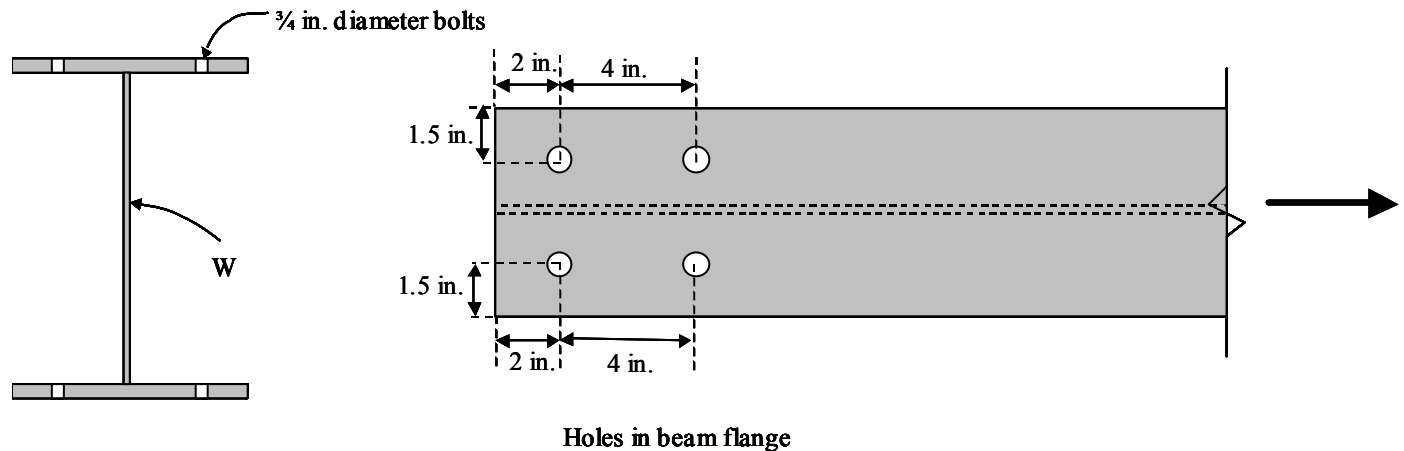
## SOLUTION

- **Step I. Select a section from the Tables**

  - Go to the **TEN** section of the AISC manual. See Table 3-1 on pages 3-17 to 3-19.

  - From this table, select W8x10 with $A_g = 2.96$ in$^2$, $A_e = 2.22$ in$^2$.

  - Gross yielding strength = 133 kips, and net section fracture strength=108 kips

  - This is the lightest section in the table.

  - Assumed $U = 0.75$. And, net section fracture will govern if $A_e < 0.923\ A_g$

- **Step II. Calculate the net section fracture strength for the actual connection**

  - According to the Figure above, $A_n = A_g - 4\ (d_b + 1/8)\ x\ t_f$

  - $A_n = 2.96 - 4\ (3/4 + 1/8)\ x\ 0.205 = 2.24$ in$^2$

  - The connection is only through the flanges. Therefore, the shear lag factor $U$ will be the distance from the top of the flange to the centroid of a WT 4 x 5.

-   See **DIM** section of the AISC manual. See Table 1-8, on pages 1-50, 1-51

-   $\bar{x} = 0.953$

-   $U = 1 - \bar{x}/L = 1 - 0.953 / 4 = 0.76$

-   $A_e = 0.76\ A_n = 0.76 \times 2.24 = 1.70\ in^2$

-   $\phi_t P_n = 0.75 \times F_u \times A_e = 0.75 \times 65 \times 1.70 = 82.9$ kips

-   <u>Unacceptable</u> because $P_u = 100$ kips; **REDESIGN** required

-   **Step III. Redesign**

    Many ways to redesign. One way is shown here:

-   Assume $\phi_t P_n > 100$ kips

-   Therefore, $0.75 \times 65 \times A_e > 100$ kips

-   Therefore, $A_e > 2.051\ in^2$

-   Assume, $A_e = 0.76\ A_n$  (based on previous calculations, step II)

-   Therefore $A_n > 2.7\ in^2$

-   But, $A_g = A_n + 4\ (d_b + 1/8) \times t_f$  (based on previous calculations, step II)

-   Therefore $A_g > 2.7 + 3.5 \times t_f$

    -   Go to the section dimension table 1-1 on page 1-22 of the AISC manual. Select next

        highest section.

        -   For W 8 x 13, $t_f = 0.255$ in.

        -   Therefore, $A_g > 2.7 + 3.5 \times 0.255 = 3.59\ in^2$

        -   From Table 1-1, W8 x 13 has $A_g = 3.84\ in^2 > 3.59\ in^2$

        -   Therefore, W8 x 13 is acceptable and is chosen.

- **Step IV. Check selected section for net section fracture**

  - $A_g = 3.84$ in$^2$

  - $A_n = 3.84 - 3.5 \times 0.255 = 2.95$ in$^2$

  - From dimensions of WT4 x 6.5, $\bar{x} = 1.03$ in.

  - Therefore, $U = 1 - \bar{x}/L = 1 - 1.03/4 = 0.74$

  - Therefore, $A_e = U\,A_n = 0.74 \times 2.95 = 2.19$ in$^2$

  - Therefore, net section fracture strength $= 0.75 \times 65 \times 2.19 = 106.7$ kips

  - Which is greater than 100 kips (design load). Therefore, W 8 x 13 is acceptable.


- **Step V. Check the block shear rupture strength**

  o Identify the block shear path
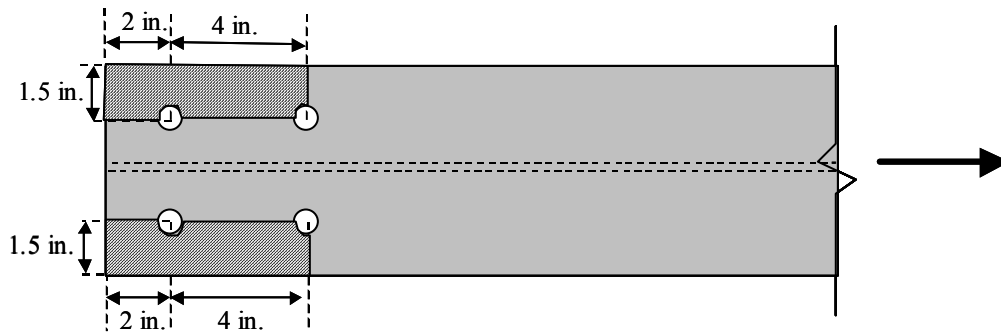


  - The block shear path is show above. **Four blocks** will separate from the tension

    member (two from each flange) as shown in the figure above.

  - $A_{gv} = [(4+2) \times t_f] \times 4 \qquad = 6 \times 0.255 \times 4 = 6.12$ in$^2$      - for four tabs

  - $A_{nv} = \{4+2 - 1.5 \times (d_b + 1/8)\} \times t_f \times 4 = 4.78$ in$^2$

  - $A_{gt} = 1.5 \times t_f \times 4 = 1.53$ in$^2$

  - $A_{nt} = \{1.5 - 0.5 \times (d_b + 1/8)\} \times t_f \times 4 = 1.084$ in$^2$

  o Identify the governing equation:

- $F_uA_{nt} = 65 \times 1.084 = 70.4$ kips

- $0.6F_uA_{nv} = 0.6 \times 65 \times 4.78 = 186.42$ kips , which is $> F_uA_{nt}$

o Calculate block shear strength

- $\phi_t R_n = 0.75 (0.6F_uA_{nv} + F_yA_{gt}) = 0.75 (186.42 + 50 \times 1.53) = 197.2$ kips

- Which is greater than $P_u = 100$ kips. Therefore W8 x 13 is still acceptable

- **Summary of solution**

| Mem. | Design load | $A_g$ | $A_n$ | U | $A_e$ | Yield strength | Fracture strength | Block-shear strength |
|------|-------------|-------|-------|------|-------|----------------|-------------------|----------------------|
| W8x13 | 100 kips | 3.84 | 2.95 | 0.74 | 2.19 | 173 kips | 106.7 kips | 197.2 kips |
| | | Design strength = 106.7 kips (net section fracture governs) | | | | | | |
| | | W8 x 13 is adequate for $P_u = 100$ kips and the given connection | | | | | | |

**EXAMPLE 3.11** Design a member to carry a factored maximum tension load of 100 kips.

(b) The member is a single angle section connected through one leg using four 1 in. diameter bolts. The center-to-center distance of the bolts is 3 in. The edge distances are 2 in. Steel material is A36
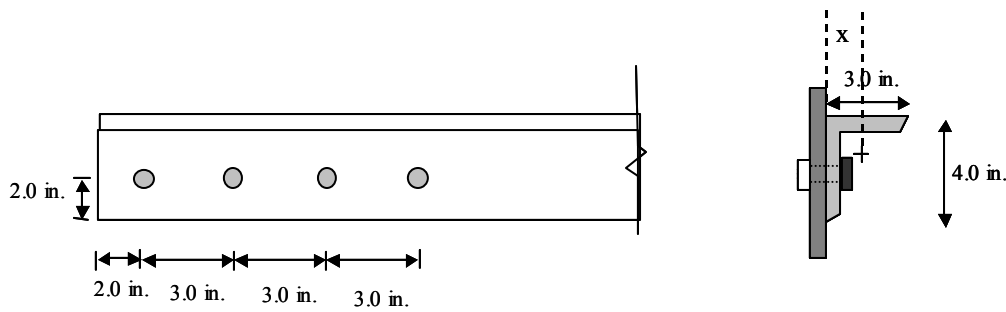


- **Step I. Select a section from the Tables**

  - Go to the **TEN** section of the AISC manual. See Table 3-2 on pages 3-20 to 3-21.

  - From this table, select $L4x3x1/2$ with $A_g = 3.25$ in$^2$, $A_e = 2.44$ in$^2$.

  - Gross yielding strength = 105 kips, and net section fracture strength=106 kips

  - This is the lightest section in the table.

  - Assumed $U = 0.75$. And, net section fracture will govern if $A_e < 0.745\ A_g$

- **Step II. Calculate the net section fracture strength for the actual connection**

  - According to the Figure above, $A_n = A_g - 1\ (d_b + 1/8)$ x t

  - $A_n = 3.25 - 1(1 + 1/8)$ x $0.5 = 2.6875$ in$^2$

  - The connection is only through the **long leg**. Therefore, the shear lag factor $U$ will be the distance from the back of the long leg to the centroid of the angle.

  - See **DIM** section of the AISC manual. See Table 1-7, on pages 1-36, 1-37

  - $\overline{x} = 0.822$ in.

  - $U = 1 - \overline{x}/L = 1 - 0.822\ /9\ = 0.908$

  - But $U$ must be $\leq 0.90$. Therefore, let $U = 0.90$

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- $A_e = 0.90\ A_n = 0.90\ \text{x}\ 2.6875 = 2.41\ \text{in}^2$

- $\phi_t P_n = 0.75\ \text{x}\ F_u\ \text{x}\ A_e = 0.75\ \text{x}\ 58\ \text{x}\ 2.41 = 104.8\ \text{kips}$

- <u>Acceptable </u>because $P_u = 100\ \text{kips}$.

- **Step V. Check the block shear rupture strength**

  o Identify the block shear path



- $A_{gv} = (9+2)\ \text{x}\ 0.5\ = 5.5\ \text{in}^2$

- $A_{nv} = [11 - 3.5\ \text{x}\ (1+1/8)]\ \text{x}\ 0.5 = 3.53\ \text{in}^2$
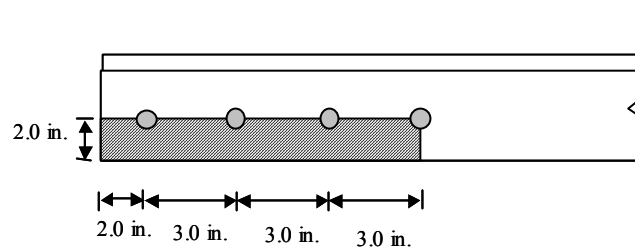
- $A_{gt} = 2.0\ \text{x}\ 0.5 = 1.0\ \text{in}^2$

- $A_{nt} = [2.0 - 0.5\ \text{x}\ (1 + 1/8)]\ \text{x}\ 0.5 = 0.72\ \text{in}^2$

  o Identify the governing equation:

- $F_u A_{nt} = 58\ \text{x}\ 0.72 = 41.76\ \text{kips}$

- $0.6 F_u A_{nv} = 0.6\ \text{x}\ 58\ \text{x}\ 3.53 = 122.844\ \text{in}^2$, which is $> F_u A_{nt}$

  o Calculate block shear strength

- $\phi_t R_n = 0.75\ (0.6 F_u A_{nv} + F_y A_{gt}) = 0.75\ (122.84 + 36\ \text{x}\ 1.0) = 119.133\ \text{kips}$

- Which is greater than $P_u = 100\ \text{kips}$. Therefore $L4\text{x}3\text{x}1/2$ is still acceptable